

Exam questions: TKO_3109 Advanced Algorithms

(answers in english)

12-February-2018

IMPORTANT: answer 4 out of the following 5 questions.

(1) (10p) In the *Subset Sum* problem we are given a set of n non-negative numbers $\{w_1, w_2, \dots, w_n\}$, and a bound W . The goal is to select a subset S of these numbers so that $\sum_{i \in S} w_i \leq W$ and $\sum_{i \in S} w_i$ is maximized. Write a *dynamic programming* algorithm that solves this problem.

(2) (10p) *Sequence alignment*: given two strings on the same alphabet, find the minimum cost alignment between these using a dynamic programming algorithm. The penalty of aligning a symbol with a gap is $\delta > 0$ and for aligning two different symbols x_i and y_i is $\alpha_{x_i y_i}$.

(3) (10p) Describe the *Maximum Flow Problem* and outline the Ford-Fulkerson algorithm for finding the maximum flow in the network (no need for proofs).

(4) (10p) *Bipartite Matching Problem*: given a graph $G = (V, E)$ where nodes are partitioned into two sets X and Y , so that all edges $e \in E$ have one end in X and one in Y . A matching M is a subset of edges ($M \subseteq E$) such that each node appears in at most one edge in M . Find the largest possible matching in G using network flows.

(5) (10p) *NP-completeness*: Given a graph $G = (V, E)$ and a bound k , the k -coloring problem asks if the vertices of the graph can be colored with at most k colors without conflict (adjacent nodes are not colored with the same color). Prove that deciding if a graph is 3-colorable is NP-complete.