

Exam questions: TKO_3109 Advanced Algorithms

(answers in english)

15-January-2018

IMPORTANT: answer 4 out of the following 5 questions.

(1) (10p) Given a graph $G = (V, E)$ with edge costs c_e associated with each edge $e \in E$. The edge costs are allowed to be negative; we assume that there are no negative cycles in the graph. Give a *dynamic programming* algorithm that finds the length of the shortest path from any node to a target node $t \in V$. How to recover an actual shortest path from the resulting table?

(2) (10p) Describe the *Maximum Flow Problem* and outline the Ford-Fulkerson algorithm for finding the maximum flow in the network (no need for proofs).

(3) (10p) A set of paths in a graph $G = (V, E)$ are said to be *edge disjoint*, if their edge sets are disjoint. The *edge disjoint paths* problem is to find the maximum number of edge disjoint paths from node $s \in V$ to node $t \in V$. Give an algorithm for this problem using flow networks. Prove that the resulting paths are edge disjoint, and their number is the maximum possible.

(4) (10p) *NP-Completeness*: The *Independent Set Problem* asks if there is an independent set of size at least k in a graph $G = (V, E)$. A subset of vertices $S \subseteq V$ is independent if no two vertices of S are joined by an edge from E . Prove that $3\text{-SAT} \leq_p \text{Independent Set}$.

(5) (10p) *NP-Completeness*: Show that the *Hamiltonian Cycle* problem is NP-Complete.