Exam questions: TKO_3109 Advanced Algorithms

(answers in english)

19-March-2018

IMPORTANT: answer 4 out of the following 5 questions.

(1) (10p) In the *Subset Sum* problem we are given a set of n non-negative numbers $\{w_1, w_2, ..., w_n\}$, and a bound W. The goal is to select a subset S of these numbers so that $\sum_{i \in S} w_i \leq W$ and $\sum_{i \in S} w_i$ is maximized. Write a *dynamic programming* algorithm that solves this problem.

(2) (10p) Given a graph $G = (V, E)$ with edge costs c_e associated with each edge $e \in E$. The edge costs are allowed to be negative; but we assume that there are no negative cycles in the graph. Give a *dynamic programming* algorithm that finds the length of the shortest path from any node to a target node $t \in V$. How to recover an actual shortest path from the resulting table?

(3) (10p) Describe the *Maximum Flow Problem* and outline the Ford-Fulkerson algorithm for finding the maximum flow in the network (no need for proofs).

(4) (10p) A set of paths in a graph $G = (V, E)$ are said to be edge disjoint, if their edge sets are disjoint. The *edge disjoint paths* problem is to find the maximum number of edge disjoint paths from node $s \in V$ to node $t \in V$. Give an algorithm for this problem using flow networks. Prove that the resulting paths are edge disjoint, and their number is the maximum possible.

(5) (10p) *NP-completeness:* Given a graph $G = (V, E)$ and a bound k, the k-coloring problem asks if the vertices of the graph can be colored with at most k colors without conflict (adjacent nodes are not colored with the same color). Prove that deciding if a graph is 3-colorable is NP-complete.