Exam questions: TKO 3108 Algorithm Design

(answers in english)

28-November-2016

(1) (8p) The *Heap* data structure has the property that for every element v, if the parent of v is w then $key(w) < key(v)$.

    (a)(3p) Describe how the heap data structure is stored in an array.

    (b)(5p) Write an algorithm that adds a new item into the heap so that the heap property is kept.

(2) (8p) Given a graph $G = (V, E)$, when is G a *Bipartite Graph* (write a definition) and how to test for *bipartiteness* (write an algorithm).

(3) (8p) Given a directed graph $G = (V, E)$, give an algorithm that constructs a *topological ordering* of the graph. What is the condition for a topological ordering to exist in G?

(4) (8p) *Minimum Spanning Trees:*

    (a)(2p) What is a *minimum spanning tree* in a graph $G = (V, E)$ with costs c_e on the edges.

    (b)(3p) Give one algorithm that outputs a minimum spanning tree (hint: Kruskal's, Prim's or Reverse Delete).

    (c)(3p) Prove that the algorithm outputs a spanning tree of minimum total cost.

(5) (8p) *Sorting:* Given an array of n numbers, give an algorithm that sorts these numbers in increasing order and runs in $O(n \log n)$ time. Prove that the asymptotic running time of the algorithm is $O(n \log n)$.