

Exam questions: TKO 5500 Design and Analysis of Algorithms

(answers in english)

10-Feb-2014

(1) (8p) *Interval Scheduling*: given a set of (job) requests $\{1, 2, \dots, n\}$, where each request has a start $s(i)$ and a finish $f(i)$ time. Two requests are compatible if their time interval do not overlap. The Interval Scheduling Problem asks to select the maximum number of compatible requests.

(a) Give a greedy algorithm that selects the maximum number of compatible requests.

(b) Show that the algorithm outputs an optimal interval scheduling.

(2) (8p) *Sorting*: Given an array of n numbers, write an algorithm that sorts these numbers in increasing order. The algorithm must run in $O(n \log n)$ time. *Mergesort*

(3) (8p) In the *Subset Sum* problem we are given a set of n non-negative numbers $\{w_1, w_2, \dots, w_n\}$, and a bound W . The goal is to select a subset S of these numbers so that $\sum_{i \in S} w_i \leq W$ and $\sum_{i \in S} w_i$ is maximized.

(a) Write a *dynamic programming* algorithm that solves this problem.

(b) What is the running time of this algorithm?

(4) (8p) Describe the *Maximum Flow Problem* and outline the Ford-Fulkerson algorithm for finding the maximum flow in a flow network.

(5) (8p) Given a graph $G = (V, E)$, the *Hamiltonian Cycle* problem asks if there is a cycle in G that visits each node exactly once and returns to the starting node, using only edges from E . Show that the *Hamiltonian Cycle* problem is NP-Complete.