ETT_2026 Digitaalinen Signaalinkäsittely Exam

April 03, 2017

1. Describe the following. **(3 p)**
   a) Linear time-invariant system and its properties.
   b) What are the uses of filters and in what type of applications they are needed?
   c) Give list of applications where adaptive signal processing is used and what are its benefits compared to non-adaptive signal processing?
2. Design a lowpass FIR filter that satisfy the given specifications using window based design method: $\omega_s = 0.3\pi$, $A_s = 50$ dB, $\omega_p = 0.5\pi$, and $A_p = 0.001$ dB. **(8 pt)**
   a) Use an appropriate fixed window to obtain a minimum order linear-phase filter and determine the coefficients of the impulse response of the filter and plot it.
   b) What will be the order of the filter if it uses Kaiser window?
3. Explain briefly the idea of multirate signal processing **(3 pt)**
   a) Describe the process of decimation and interpolation.
   b) Why decimation and interpolation are required in filter banks?
   c) What are the benefits of multrate signal processing and give example applications where multirate signal processing is applied.
4. Consider the filter shown in Figure 1. **(8 pt)**
   a) Determine its system function
   b) Sketch the pole-zero plot and check for stability if 1) b0 = b2 = 1, b1 = 2, a1 = 1.5, a2 = -0.9; 2) b0 = b2 = 1, b1 = 2, a1 = 1, a2 = -2;
   c) Calculate the output of the system when the input is $x(n) = 3 + \sin(0.5\pi n)$ and b0 = b2 = 1, b1 = 2, a1 = 1, a2 = -2;

5. The impulse response of a system is given as follows $H(z) = 1-2z^{-1}+3z^{-2}-4z^{-3}+4z^{-5}$ **(8 pt)**
   a) Determine the corresponding frequency response at frequency $\omega = 0.05*2\pi$.
   b) Determine the amplitude and phase response at frequency $\omega = 0.05*2\pi$.
   c) What is the system output response if the system input $x(z) = = z^{-10}/1-z^{-1}$?

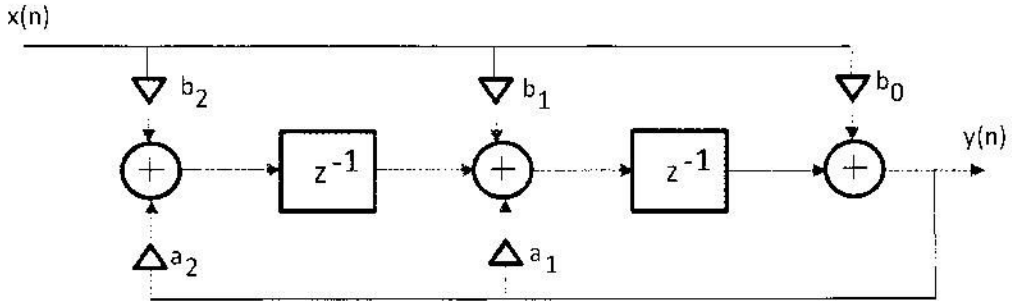

Figure 1: direct form filter for question 4

N DFT $X(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}, \quad for\ k = 0, 1, 2, \ldots, N-1$

Amplitude spectrum $A_k = \frac{1}{N}|X(k)| = \frac{1}{N}\sqrt{(Real[X(k)])^2 + (Imag[X(k)])^2}$

Phase spectrum $\varphi_k = tan^{-1}(\frac{Imag[X(k)]}{Real[X(k)]})$

Power spectrum $P_k = \frac{1}{N^2}|X(k)|^2$

Twiddle matrix $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

Highpass filter impulse response

$$h_{hp}(n) = \frac{\sin[\pi(n-0.5M)]}{\pi(n-0.5M)} - \frac{\sin[\omega_c(n-0.5M)]}{\pi(n-0.5M)}$$

Bandpass filter impulse response

$$h_{bp}(n) = \frac{\sin[\omega_{c2}(n-0.5M)]}{\pi(n-0.5M)} - \frac{\sin[\omega_{c1}(n-0.5M)]}{\pi(n-0.5M)}$$

Lowpass filter impulse response

$$h_{lp}(n) = \frac{\sin[\omega_c(n-0.5M)]}{\pi(n-0.5M)}$$

Table 10.3 Properties of commonly used windows ($L = M + 1$).

Window name	Side lobe level (dB)	Approx. $\Delta\omega$	Exact $\Delta\omega$	$\delta_p \approx \delta_s$	A_p (dB)	A_s (dB)
Rectangular	−13	$4\pi/L$	$1.8\pi/L$	0.09	0.75	21
Bartlett	−25	$8\pi/L$	$6.1\pi/L$	0.05	0.45	26
Hann	−31	$8\pi/L$	$6.2\pi/L$	0.0063	0.055	44
Hamming	−41	$8\pi/L$	$6.6\pi/L$	0.0022	0.019	53
Blackman	−57	$12\pi/L$	$11\pi/L$	0.0002	0.0017	74

Bartlett (triangular)

$$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2, \; M \text{ even} \\ 2 - 2n/M, & M/2 < n \le M \\ 0, & \text{otherwise} \end{cases}$$

Hann

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Kaiser

$$M = \frac{A - 8}{2.285\,\Delta\omega}, \qquad \beta = \begin{cases} 0, & A < 21 \\ 0.5842(A-21)^{0.4} + 0.07886(A-21), & 21 \le A \le 50 \\ 0.1102(A - 8.7), & A > 50 \end{cases}$$