

Exam questions: TKO 3108 Algorithm Design

28.1.2019

(answers in english)

(1) (8p) *Breadth-First Search*: given a graph $G = (V, E)$ and a starting node s . Describe the Breadth-First Search (BFS) traversal of G starting from node s . What is the connected component of s , and how to use BFS to determine if two nodes s and t are connected.

(2) (8p) Given a directed graph $G = (V, E)$, give an algorithm that constructs a *topological ordering* of the graph. What is the condition for a topological ordering to exist in G ?

(3) (8p) *Shortest paths*: Given a graph $G = (V, E)$ with non-negative edge lengths l_e for each edge $e \in E$.

(a) What is Dijkstra's algorithm to find the shortest path from a node s to all other nodes and how it works.

(b) How to implement Dijkstra's algorithm to run in $O(m \log n)$ time.

(c) Prove that Dijkstra's algorithm outputs the shortest path.

(4) (8p) *Sorting*: Given an array of n numbers, give an algorithm that sorts these numbers in increasing order and runs in $O(n \log n)$ time. Prove that the asymptotic running time of the algorithm is $O(n \log n)$.

(5) (8p) *Counting inversions*: Given an array of numbers in arbitrary order, count the number of inversions using an algorithm. A pair is an inversion if the larger number occurs before the smaller one in the array. Write a recursive algorithm that outputs the number of inversions. What is the running time of this algorithm?