ETT_2026 Digitaalinen Signaalinkäsittely Exam

March 04, 2019

Students are allowed to use only calculator. The necessary equations and references are given in the last two pages of this exam paper.

1. Describe the following. **(5 p)**
   a) The applications of digital signal processing.
   b) Specify some of the basic purposes of filters and give type of applications where the filters are needed.
   c) Give list of applications where adaptive signal processing is used and what are its benefits compared to non-adaptive signal processing?
   d) The applications of two-dimensional filters.
   e) What are the benefits of multrate signal processing and give example applications where multirate signal processing is applied.

2. Design a lowpass FIR filter that satisfy the given specifications using window based design method: $\omega_p = 0.3\pi$, $A_s = 50$ dB, $\omega_s = 0.5\pi$, and $A_p = 0.1$ dB. **(8 pt)**
   a) Use an appropriate fixed window to obtain a minimum order linear-phase filter and determine the coefficients of the impulse response of the filter and plot it.
   b) What will be the order of the filter if it uses Kaiser window?

3. The impulse response of a system is given as follows H(z) = $z^{-4} - z^{-2} + 3z + 1$ **(4 pt)**
   a) If the input is x(z) = $1/1\text{-}z^{-1}$, what is the output in frequency domain?
   b) Determine the corresponding frequency response of the system at frequency $\omega = 0.25*2\pi$.

4. The system function of a discrete-time LTI system is as follows **(8 pt)**

$$H(z) = \frac{z^2 + 3z + 1}{z^2 + 0.3z + 0.8}$$

   a) Determine the time-domain difference equation of the system.
   b) Draw direct form II structure of the system.
   c) Calculate the output of the system when the input is $x(n) = 3 + \cos(0.5\pi n)$
   d) Is the system stable, why?

N DFT $(k) = \sum_{n=0}^{N-1} x(n) W_N^{nk}, \quad for\ k = 0, 1, 2, \ldots, N-1$

Amplitude spectrum $A_k = \frac{1}{N}|X(k)| = \frac{1}{N}\sqrt{(Real[X(k)])^2 + (Imag[X(k)])^2}$

Phase spectrum $\varphi_k = tan^{-1}(\frac{Imag[X(k)]}{Real[X(k)]})$

Power spectrum $P_k = \frac{1}{N^2}|X(k)|^2$

Twiddle matrix $W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}$

$$A_p = 20 \log_{10}\left(\frac{1+\delta_p}{1-\delta_p}\right) > 0(\approx 0)$$

$$A_s = 20 \log_{10}\left(\frac{1+\delta_p}{\delta_s}\right) > 0(\gg 1)$$

$$\delta_p = \frac{10^{A_p/20} - 1}{10^{A_p/20} + 1}$$

$$\delta_s = \frac{1+\delta_p}{10^{A_s/20}}$$

Lowpass filter impulse response

$$h_{lp}(n) = \frac{\sin[\omega_c(n - 0.5M)]}{\pi(n - 0.5M)}$$

Highpass filter impulse response

$$h_{hp}(n) = \frac{\sin[\pi(n - 0.5M)]}{\pi(n - 0.5M)} - \frac{\sin[\omega_c(n - 0.5M)]}{\pi(n - 0.5M)}$$

Bandpass filter impulse response

$$h_{bp}(n) = \frac{\sin[\omega_{c2}(n - 0.5M)]}{\pi(n - 0.5M)} - \frac{\sin[\omega_{c1}(n - 0.5M)]}{\pi(n - 0.5M)}$$

Table 10.3 Properties of commonly used windows ($L = M + 1$).

Window name	Side lobe level (dB)	Approx. $\Delta\omega$	Exact $\Delta\omega$	$\delta_p \approx \delta_s$	A_p (dB)	A_s (dB)
Rectangular	−13	$4\pi/L$	$1.8\pi/L$	0.09	0.75	21
Bartlett	−25	$8\pi/L$	$6.1\pi/L$	0.05	0.45	26
Hann	−31	$8\pi/L$	$6.2\pi/L$	0.0063	0.055	44
Hamming	−41	$8\pi/L$	$6.6\pi/L$	0.0022	0.019	53
Blackman	−57	$12\pi/L$	$11\pi/L$	0.0002	0.0017	74

Bartlett (triangular)

$$w[n] = \begin{cases} 2n/M, & 0 \le n \le M/2, \quad M \text{ even} \\ 2 - 2n/M, & M/2 < n \le M \\ 0, & \text{otherwise} \end{cases}$$

Hann

$$w[n] = \begin{cases} 0.5 - 0.5\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Hamming

$$w[n] = \begin{cases} 0.54 - 0.46\cos(2\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Blackman

$$w[n] = \begin{cases} 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M), & 0 \le n \le M \\ 0, & \text{otherwise} \end{cases}$$

Kaiser

$$M = \frac{A - 8}{2.285\Delta\omega}, \qquad \beta = \begin{cases} 0, & A < 21 \\ 0.5842(A - 21)^{0.4} + 0.07886(A - 21), & 21 \le A \le 50 \\ 0.1102(A - 8.7), & A > 50 \end{cases}$$