

# Exam questions: TKO 3108 Algorithm Design

(answers in english)

26-November-2018

(1) (8p) *Breadth-First Search*: given a graph  $G = (V, E)$  and a starting node  $s$ . Describe the Breadth-First Search (BFS) traversal of  $G$  starting from node  $s$ . What is the connected component of  $s$ , and how to use BFS to determine if two nodes  $s$  and  $t$  are connected.

(2) (8p) Given a directed graph  $G = (V, E)$ , give an algorithm that constructs a *topological ordering* of the graph. What is the condition for a topological ordering to exist in  $G$ ?  $v_i, v_j$  where  $i < j$   
start with empty set

(3) (8p) *Shortest paths*: Given a graph  $G = (V, E)$  with non-negative edge lengths  $l_e$  for each edge  $e \in E$ .  
compare edge costs

(a) What is Dijkstra's algorithm to find the shortest path from a node  $s$  to all other nodes and how it works.

(b) How to implement Dijkstra's algorithm to run in  $O(m \log n)$  time.

(c) Prove that Dijkstra's algorithm outputs the shortest path.

(4) (8p) *Sorting*: Given an array of  $n$  numbers, give an algorithm that sorts these numbers in increasing order and runs in  $O(n \log n)$  time. Prove that the asymptotic running time of the algorithm is  $O(n \log n)$ .

(5) (8p) *Counting inversions*: Given an array of numbers in arbitrary order, count the number of inversions using an algorithm. A pair is an inversion if the larger number occurs before the smaller one in the array. Write a recursive algorithm that outputs the number of inversions. What is the running time of this algorithm?