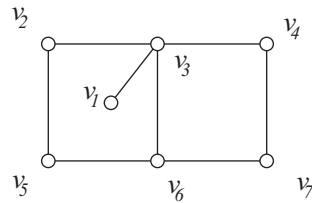


Graph Theory

Exam 4.3.2021

The exam time is 3 hours.

1. Is the sequence 7, 7, 5, 4, 4, 4, 4, 4, 4, 1 graphical?
2. a) Show that $\dim(G) = 2$ for the following graph G :



- b) Let G be a graph. Explain why the following bound holds:

$$\chi'(G) \geq \frac{|E(G)|}{\alpha'(G)}.$$

3. a) Let $k \geq 2$. For which values of the parameter k can the set of edges of an odd graph O_k be partitioned into cycles (that is, an edge belongs to a unique cycle, but the vertices can belong to several cycles)?
- b) Assume that G is a connected and bipartite graph of order 140. Can the graph G be planar, if the size of G is 193 and $g(G) \geq 7$?
4. a) Suppose that the order of a graph is 242 and the minimum degree equals 130. Show that the graph has a perfect matching.
- b) Let G be a graph with $\delta(G) \geq 2$. Assume also that $g(G)$ is divisible by three and $g(G) \geq 6$. Prove that

$$\gamma(G) \leq \frac{|V| - g(G)}{2} + \frac{g(G)}{3}.$$